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Nondiffracting electromagnetic fields in inhomogeneous isotropic media

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Abstract

In this paper stationary nondiffracting vector fields are discussed. Using the definition of propagating and evanescent nondiffracting waves based on the invariance of the intensity distribution during propagation, all the types of such fields in stratified inhomogeneous media are found.

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The search for new nondiffracting beams and physical foundations of their existence seems to be an exciting problem of modern theoretical and experimental physics. Such light constructions are believed to have a number of advantages in comparison with usual beams and pulses. These properties can be used in some optical scientific setups or in industrial devices.

First of all we need to note that diffraction is a phenomenon associated with the wave nature of light. This causes the light to spread. We can easily make certain of this if we consider, for example, the Gaussian beam widely used to focus light. However, it turns out that it is possible to overcome diffraction using the so-called nondiffracting beams.

The simplest nondiffracting light construction is the plane wave. Other, more realistic, nondiffracting solutions of the wave equation in homogeneous media are usually represented as a superposition of plane waves with wave vectors lying on the cone. The first and most studied kind of such conical beams is the Bessel beam [1]. It was obtained experimentally by Durnin *et al* in [2] and now is usually implemented by using axicon, holographic techniques, or a Fabry–Perot cavity. In contrast to the ideal mathematical construct of [1], it was shown that it is possible to make only an approximation to Bessel beams (quasi-Bessel beams) [2], which possess the properties of the Bessel beam only over a finite distance. These properties include the presence of a central core (bright or dark as in the case of the high-order Bessel beam [3]) and self-reconstruction [4]; they can carry orbital angular momentum [5] and optical vortices [6]. All these properties lead to a number of practical uses of Bessel beams, such as optical manipulation of particles, cooling and transport of gas particles, especially to achieve Bose–Einstein condensation, second harmonic generation, and so on [7–9]. A more rigorous and extensive review of the properties and applications of Bessel beams is given in [10]. Except for the Bessel beam, which is a light pattern with circular symmetry, there are fundamental nondiffracting modes with different structures arising from consideration of wave equation in other coordinate systems. The solutions with elliptical symmetry can be described by Mathieu functions and they are called Mathieu beams [11]. The parabolic solution has also been recently found [12]. The main purpose of the present paper is to give general conditions of nondiffracting beam propagation without considering particular light patterns. Therefore, we base our approach on the Helmholtz equation and on manipulation with the time-averaged Poynting vector, i.e. intensity.

The evident definition of a nondiffracting field propagating along the z-axis is the wave of the form

$$\boldsymbol{E}(x, y, z) = \boldsymbol{E}(x, y)\,\mathrm{e}^{\mathrm{i}\beta z}, \tag{1}$$

where $\boldsymbol{E}$ is the electric field strength and β is the longitudinal wavenumber. More general definition based on the Poynting vector is given in papers [13, 14]. The time-averaged Poynting vector $\boldsymbol{I} = \langle \boldsymbol{S} \rangle = (c/8\pi)\,\mathrm{Re}\,[\boldsymbol{E} \times \boldsymbol{H}^*]$ can be decomposed into the longitudinal (along the z-axis) and transversal components as $\boldsymbol{I} = \boldsymbol{I}_L + \boldsymbol{I}_T$, where $\boldsymbol{H}$ is the magnetic field strength. The stationary beam is diffraction free, if

$$\mathrm{div}\boldsymbol{I}_T = 0. \tag{2}$$

Such definition has simple geometrical interpretation: the transverse component of the nondiffracting field energy flux has no source. Relationship (2) also includes all the fields with zero transverse Poynting vector $\boldsymbol{I}_T = 0$.

However, the aforementioned definition has the drawback when it is applied to evanescent beams. In fact, the energy flux of the superposition of TE- and TM-polarized evanescent Bessel beams in the homogeneous isotropic medium with dielectric permittivity ε and magnetic permeability μ equals

$$\boldsymbol{I} = \frac{c}{8\pi}\,\mathrm{e}^{-2\beta' z}\left(\frac{k\nu}{q^2 r}(\mu|A|^2 + \varepsilon|B|^2)J_\nu^2(qr)\,\mathbf{e}_\varphi - \frac{2\nu}{q^3 r}(\beta'^2 + k^2\varepsilon\mu)J_\nu'(qr)J_\nu(qr)\,\mathrm{Im}[AB^*]\,\mathbf{e}_z - \frac{2\beta'\nu}{q^2 r}J_\nu^2(qr)\,\mathrm{Im}[AB^*]\,\mathbf{e}_r\right), \tag{3}$$

where $\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{e}_z$ are the basis vectors of the cylindrical coordinates (r, φ, z), $\beta' = -\mathrm{i}\beta$, q is the radial wavenumber, $k = \omega/c$ is the wavenumber in the vacuum, ω is the wave frequency, J_ν is the Bessel function of the first kind of the order ν, $J_\nu'(x) = \mathrm{d}J_\nu/\mathrm{d}x$, the symbol $*$ denotes complex conjugate, and A and B are the amplitude factors of TE and TM beams, respectively. To get to the energy flux (3), one should take the superposition of TE and TM Bessel beams with complex coefficients A and B. Electric and magnetic fields of TE and TM Bessel beams are given in [14]. By substituting the propagation constant β for β', the fields of evanescent Bessel beams can be derived. The averaged Poynting vector for such fields equals (3).

It is obvious that such evanescent Bessel beam does not satisfy condition (2), since the energy flux has radial component. Hence, expression (3) corresponds to the diffracting beam. For $B = 0$ one obtains the energy flux of TE wave; the case $A = 0$ gives the Poynting vector of TM wave. Both TE and TM Bessel beams possess only azimuth energy flux and so are nondiffracting according to definition (2). In our opinion, this result is illogical because the superposition of two diffraction free beams with equal longitudinal wavenumbers diffracts.

To avoid incorrect statements, we will use another definition in this paper: the nondiffracting stationary vector field is the field the energy flux $\boldsymbol{I}$ of which does not change (for propagating waves) or changes uniformly (for evanescent waves). In the first case the intensity is equal to $\boldsymbol{I}(x, y, z) = \boldsymbol{I}(x, y)$, and in the second case $\boldsymbol{I}(x, y, z) = \boldsymbol{I}(x, y)g(z)$, where $g(z)$ is a scalar function. Such definition for evanescent beams implies the conservation of the intensity distribution during their motion, but the intensity itself decreases. Further, the types of nondiffracting fields in inhomogeneous isotropic media will be found.

Let us consider a vector electromagnetic field propagating in the z-direction in a nonmagnetic isotropic medium with dielectric permittivity $\varepsilon(x, y, z)$. The medium is regarded to be nonabsorbing, i.e. ε is the real function. In such a medium the wave can be diffraction free only if the dielectric permittivity depends on the longitudinal coordinate z:

$$\varepsilon(x, y, z) = \varepsilon(z). \tag{4}$$

The electric field strength satisfies the Helmholtz equation

$$\Delta \boldsymbol{E}(x, y, z) + k^2\varepsilon(z)\boldsymbol{E}(x, y, z) = 0$$

and can be divided as follows:

$$\boldsymbol{E}(x, y, z) = \boldsymbol{E}(x, y) f(z), \tag{5}$$

where $\Delta = \nabla^2$ is the Laplace operator and ∇ is the nabla operator. The functions $\boldsymbol{E}(x, y)$ and $f(z)$ can be found from the equations

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\boldsymbol{E}(x, y) + q^2\boldsymbol{E}(x, y) = 0, \tag{6}$$

$$\frac{\mathrm{d}^2 f(z)}{\mathrm{d}z^2} + (k^2\varepsilon(z) - q^2) f(z) = 0. \tag{7}$$

From here on, we denote $\boldsymbol{E}(x, y) = \boldsymbol{E}$. The solution of equation (6) can be written as the plane wave superposition with the wavenumber q. Therefore, the strength $\boldsymbol{E}$ can be expressed by means of the functions $\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r})$ (plane waves), $\sin(\boldsymbol{q}\boldsymbol{r})$ (standing waves), J_ν (Bessel beams), etc. Generally, the functions $\boldsymbol{E}$ and $f = \rho(z)\exp(\mathrm{i}\psi(z))$ are complex. Since $\varepsilon(z)$ is the real function, from equation (7) it follows the conservation law, which is expressed as

$$\mathrm{Im}\left[f\frac{\mathrm{d}f^*}{\mathrm{d}z}\right] = \alpha = \mathrm{const}. \tag{8}$$

Selecting the function $\varepsilon(z)$ one can realize different types of fields (5). The magnetic field strength is determined from the Maxwell equations; it equals

$$\boldsymbol{H}(x, y, z) = \frac{1}{\mathrm{i}k}\left(f\nabla \times \boldsymbol{E} + \frac{\mathrm{d}f}{\mathrm{d}z}\mathbf{e}_z \times \boldsymbol{E}\right).$$

Then the time-averaged Poynting vector takes the form

$$\boldsymbol{I}(x, y, z) = \frac{c}{8\pi k}\left(|f|^2\boldsymbol{Q} - \boldsymbol{P} + \left(|\boldsymbol{E}_\perp|^2\,\mathrm{Im}\left[f\frac{\mathrm{d}f^*}{\mathrm{d}z}\right] - \mathrm{Im}[\boldsymbol{E}_\perp\nabla E_z^*]|f|^2\right)\mathbf{e}_z\right), \tag{9}$$

where $\boldsymbol{E} = \boldsymbol{E}_\perp + E_z\,\mathbf{e}_z$, $\boldsymbol{Q} = \mathrm{Im}[(\mathbf{e}_z\nabla \times \boldsymbol{E}_\perp^*)\boldsymbol{E}_\perp \times \mathbf{e}_z + E_z\nabla E_z^*]$, $\boldsymbol{P} = \mathrm{Im}[f(\mathrm{d}f^*/\mathrm{d}z)E_z\boldsymbol{E}_\perp^*]$.

First the existence of the nondiffracting propagating wave will be considered. For such a vector field $\boldsymbol{I} = \boldsymbol{I}(x, y)$ should be fulfilled. From (9) one can see that the longitudinal energy flux component I_z consists of two summands. The first summand does not depend on the coordinate z owing to expression (8). The second summand does not depend on z in each of the following four cases. In each of these cases the beam is the nondiffracting one.

(i) $|f|^2 = \gamma = \text{const}$. From this condition it follows that $f(z) = \sqrt{\gamma}\exp(\mathrm{i}\beta z)$ and $\alpha = \gamma\beta$. Such diffraction free beams can propagate only in homogeneous media. The beams are characterized by the longitudinal wavenumber $\beta^2 = k^2\varepsilon - q^2$. Vector $\boldsymbol{E}$ is an arbitrary complex vector satisfying equation (6). Therefore, such beams are characterized by an arbitrary elliptical polarization.

(ii) $\boldsymbol{E}$ is a real vector, i.e. the beam polarization is linear. In this case the energy flux is equal to

$$\boldsymbol{I}(x, y) = \frac{\alpha c}{8\pi k}(|\boldsymbol{E}_\perp|^2\,\mathbf{e}_z - E_z\boldsymbol{E}_\perp)$$

for any complex function f. Function f satisfies condition (8), which determines the link between its real and imaginary parts. It is easy to find the function itself,

$$f(z) = \rho(z)\exp\left(-\mathrm{i}\alpha\int\frac{\mathrm{d}z}{\rho^2(z)}\right). \tag{10}$$

The absolute value $\rho(z)$ of f can be expressed from the differential equation (7). If one wants the function ρ to be the solution of equation (7), then he or she should take the dielectric permittivity in the form

$$\varepsilon = \frac{1}{k^2}\left(\frac{\alpha^2}{\rho^4} - \frac{1}{\rho}\frac{\mathrm{d}^2\rho}{\mathrm{d}z^2} + q^2\right). \tag{11}$$

(iii) $\boldsymbol{E}_\perp = a\,\mathbf{e}_z \times \nabla E_z^*$, where a is an arbitrary complex number. This condition causes the zero second summand in I_z. In the case under consideration $\boldsymbol{Q} \sim \mathrm{Im}[E_z\nabla E_z^*]$. The energy flux have to be z-independent; therefore, the condition $\boldsymbol{Q} = 0$ or, equivalently, $\mathrm{Im}[E_z\nabla E_z^*] = 0$ should hold true. From the last expression it follows the conclusion that the longitudinal component of the electric field E_z should be the real function of transverse coordinates. The vector

$$\boldsymbol{P} = \left(\alpha\,\mathrm{Re}\,[a] - \mathrm{Re}\left[f\frac{\mathrm{d}f^*}{\mathrm{d}z}\right]\mathrm{Im}[a]\right)E_z(\mathbf{e}_z \times \nabla E_z)$$

has to be z-independent, too. Therefore, we can write the equation $\mathrm{Re}[f(\mathrm{d}f^*/\mathrm{d}z)] = \sigma = \text{const}$, from which it follows that $\rho = \sqrt{\sigma z + C}$, where C is a constant. For this function ρ one can find f and dielectric permittivity from expressions (10) and (11), respectively. Therefore, there is an elliptically polarized nondiffracting field which is described by the vector complex wave

$$\boldsymbol{E}(x, y, z) = (a\,\mathbf{e}_z \times \nabla E_z + E_z\,\mathbf{e}_z)(\sigma z + C)^{1/2-\mathrm{i}\alpha/\sigma} \tag{12}$$

propagating in the inhomogeneous medium

$$\varepsilon = \frac{\sigma^2 + 4\alpha^2}{4k^2(\sigma z + C)^2} + \frac{q^2}{k^2}. \tag{13}$$

The medium (13) is frequency dispersive because the dielectric permittivity includes the wavenumber k.

(iv) $\boldsymbol{E}_\perp = b\nabla E_z$, where b is a real number. Then the second summand in the longitudinal energy flux component I_z becomes zero. The vector $\boldsymbol{Q} = \mathrm{Im}[E_z\nabla E_z^*]$ should become zero, too. Therefore, E_z and $\boldsymbol{E}$ are real quantities, i.e. this case is the particular case of (ii).

According to our definition an evanescent wave is diffraction free when the energy flux can be presented in the form $\boldsymbol{I}(x, y, z) = \boldsymbol{I}(x, y)g(z)$. For evanescent waves, the function $f(z)$ is real and so $\alpha = 0$ and the Poynting vector (9) equals

$$\boldsymbol{I}(x, y, z) = \frac{c}{8\pi k}\left(f^2\boldsymbol{Q} - f\frac{\mathrm{d}f}{\mathrm{d}z}\,\mathrm{Im}[E_z\boldsymbol{E}_\perp^*] - f^2\,\mathrm{Im}[\boldsymbol{E}_\perp\nabla E_z^*]\,\mathbf{e}_z\right). \qquad (14)$$

The wave will be nondiffracting in the following cases.

(v) $f(\mathrm{d}f/\mathrm{d}z) = mf^2$, where m is a constant. Then $f = \exp(mz)$ and $g(z) = \exp(2mz)$. The considered solutions correspond to the waves in homogeneous media, for which $m = -\beta'$. The vector Bessel beam with the energy flux (3) is the typical example of such a field.

(vi) $\mathrm{Im}[E_z\boldsymbol{E}_\perp^*] = 0$; then $g(z) = f^2$. The function f can be found from (7) for any inhomogeneous medium. Then vector $\boldsymbol{E}$ can be an arbitrary real vector satisfying (6) or a complex vector of the form $\boldsymbol{E} = E_z\boldsymbol{a}$, where $\boldsymbol{a}$ is a real constant vector.

(vii) $\mathrm{Im}[\boldsymbol{E}_\perp\nabla E_z^*] = 0, \boldsymbol{Q} = 0$; then $g(z) = f(\mathrm{d}f/\mathrm{d}z)$. Such conditions correspond to items (ii)–(iv) for propagating waves. For real vectors $\boldsymbol{E}$, the energy flux is equal to zero and evanescent beams are diffraction free. In case (iii), when the vector field equals $\boldsymbol{E}(x, y) = a\,\mathbf{e}_z \times \nabla E_z + E_z\,\mathbf{e}_z$, the Poynting vector of the evanescent wave takes the form

$$\boldsymbol{I} = \frac{c}{8\pi k}f\frac{\mathrm{d}f}{\mathrm{d}z}\,\mathrm{Im}[a]E_z\,\mathbf{e}_z \times \nabla E_z.$$

In contrast to propagating waves, the just considered evanescent beams are nondiffracting in any z-inhomogeneous medium.

Thus, the following vector fields (5) are diffraction free: elliptically polarized waves with complex $\boldsymbol{E}$ and $f = \exp(\mathrm{i}\beta z)$ for propagating waves or $f = \exp(-\beta' z)$ for evanescent waves in a homogeneous medium; propagating and evanescent linearly polarized waves with real $\boldsymbol{E}$ and function f determined from (10) in an arbitrary z-inhomogeneous medium; propagating elliptically polarized waves of the form (12) in the medium (13); and evanescent waves with complex vectors $\boldsymbol{E} = E_z\boldsymbol{a}$ ($\boldsymbol{a}$ is a real constant vector) and $\boldsymbol{E} = a\,\mathbf{e}_z \times \nabla E_z + E_z\,\mathbf{e}_z$ (E_z is a real function) in any z-inhomogeneous medium.

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